Principles of Excluded Middle and Contradiction

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Abstract:

Peirce’s principles of excluded middle and contradiction more resembled those of Aristotle than those of contemporary logicians. While the principles themselves are simple and straightforward, many of Peirce’s comments about them have been misunderstood by commentators. In particular, his belief that the principle of excluded middle does not apply to the general (or to propositions expressing necessity) and that the principle of contradiction does not apply to the vague (or to propositions expressing possibility) have been mistakenly connected to his eventual rejection of the principle of bivalence and development of three-valued logical connectives. An understanding of Peirce’s view of those logical principles shows that those beliefs motivated neither his rejection of bivalence nor his work in triadic logic.

Keywords: Excluded Middle, Contradiction, Logic, Bivalence, Modality

The Principles

From a contemporary point of view, Peirce’s conception of the principles of excluded middle and contradiction might seem non-standard. By “principle of excluded middle,” he did not mean:

> **Law of Excluded Middle (LEM)**
> Every instance of “p or not-p” is true.[1]
> p V ~p[2]
> Either p or not-p.[3]

And by “principle of contradiction” he did not mean:

> **Law of Non-Contradiction (LNC)**
> Every instance of “p and not-p” is false.[4]
> ~(p & ~p)[2]
> Not both p and not-p.[3]

Rather, Peirce’s formulations of those principles resembled Aristotle’s.[5] Peirce sometimes expressed the principles in the material mode, sometimes in the formal mode. As he understood them, the principles are as follows:

Principle of Excluded Middle (PEM)

Material mode: for any property and for any individual, either that individual possesses >that property or that individual does not possess that property.

Formal mode: for any pair of contradictory predicates "P" and "not-P" and for any >individual (non-general) subject-term "S", either "S is P" or "S is not-P" is true.[6]

Principle of Contradiction (PC)

Material mode: for any property and for any definite subject, it is not the case both that >>the subject possesses that property and that the subject does not possess that property.

Formal mode: for any pair of contradictory predicates "P" and "not-P" and for any definite subject-term "S", "S is P" and "S is not-P" are not both true.[7]

The Principles and Peirce's "Logic of Vagueness"

Peirce's principles are crucial to a correct understanding of his so-called "logic of vagueness" (LOV) (5.506, c.1905), his account of the various sorts of indeterminacy which can affect the meaning of a sign (see Chiasson, 2001). The LOV consists, in part, of Peirce's views on the meaning of propositional subject-terms. On Peirce's view, a propositional subject-term (or terms), which refers to the proposition's object (or objects), can be determinate or indeterminate. A determinate propositional subject-term, e.g., a proper name or a definite description, is one which picks out a definite individual to which the predicate is purported to apply; Peirce often called these *singular* subjects (e.g., 5.152ff., 1903). An indeterminate propositional subject is one which does not pick out a definite individual. I will refer to propositions with determinate (singular) subjects as object-*determinate* and to propositions with indeterminate subjects as *object-indeterminate*.

There are two main types of object-indeterminacy: generality (or universality) and vagueness (or indefiniteness) (5.447-9, 1905). General object-indeterminacy is, roughly, universal quantification; and vague, or indefinite, object-indeterminacy is, roughly, existential quantification.[8] We need to be especially careful not to misunderstand what Peirce meant by "vagueness" in this context. He did not mean what most philosophers now use the word to mean, viz. the property of having cases of indeterminate or borderline application (i.e., the property commonly called "fuzziness"). To avoid confusion, I will avoid using the word "vagueness" to refer to the type of object-indeterminacy other than generality. Since Peirce frequently used "definite" to refer to the opposite of this sort of vagueness, I will use "indefiniteness" rather than

"vagueness" to refer to the type of object-indeterminacy other than generality.[9] So, Peirce's view was that all propositional subject-terms can be sorted into three categories: the general, the indefinite (or vague), and the determinate (or singular).

The relevance of all this to the principles of excluded middle and contradiction is as follows. Peirce wrote that "anything is *general* in so far as the principle of excluded middle does not apply to it," e.g., the proposition "Man is mortal," and that "anything" is indefinite "in so far as the principle of contradiction does not apply to it," e.g., the proposition "A man whom I could mention seems to be a little conceited" (5.447-8, 1905). If we take Peirce to have meant LEM and LNC, then it appears that he wanted to deny the principle of bivalence (according to which all propositions are true or else false) with regard to universally quantified propositions, and that he meant to claim that existentially quantified propositions are both true and false. But why think that "Man is mortal," which seems to be straightforwardly true, is *neither* true nor false? And why think that one and the same proposition, "A man whom I could mention seems to be a little conceited," is *both* true and false? Once we see what Peirce meant by "principles of excluded middle and contradiction," we see that this is not what he was claiming.

Peirce's PEM is a principle about individual subjects. Specifically, it gives a necessary condition of individuality: (in the material mode) if S is an individual, then, for any property P, either S is P or S is not-P; or (in the formal mode) if "S" is an individual subject-term, then, for any predicate "P", either "S is P" is true or "S is not-P" is true.[10] So PEM (formal mode) is equivalent to the claim that for any individual (non-general) subject term "S" and for any predicate "P", the proposition "S is P or S is not-P" is true. Peirce said that PEM does not apply to the general because it is not the case, with regard to every predicate "P" and every general subject-term "S", that "S is P or S is not-P" is true; sometimes such propositions are false (e.g., "All Floridians live in Palm Beach County or all Floridians do not live in Palm Beach County"). So Peirce's claim that PEM does not apply to the general does not imply that general propositions are neither true nor false.

Similarly, Peirce's PC is a principle *about definite subjects*. Specifically, it gives a necessary condition of definiteness: (in the material mode) if S is definite, then S is not both P and not-P, or (in the formal mode) if "S" is a definite subject-term, then "S is P" and "S is not-P" are not both true. So PC (formal mode) is equivalent to the claim that for any definite (not indefinite) subject term "S" and for any predicate "P", the proposition "S is P and S is not-P" is false. Peirce says that PC does not apply to the indefinite because it is not the case, with regard to every predicate "P" and every

indefinite subject-term “S”, that “S is P and S is not-P” is false; sometimes such propositions are true (e.g., “Some philosophers own dogs and some philosophers do not own dogs”). So Peirce’s claim that PC does not apply to the indefinite (vague) does not imply that indefinite (vague) propositions are both true and false.[11]

Modal Propositions

Peirce also denied the applicability of PEM and PC to modal propositions. He held that PEM does not apply to propositions that express necessity (e.g., “S must be P”) and that PC does not apply to propositions that express possibility (e.g., “S can be P” and “S may be P”):[12]

> … that which characterizes and defines an assertion of Possibility is its emancipation from the Principle of Contradiction, while it remains subject to the Principle of Excluded Third; while that which characterizes and defines an assertion of Necessity is that it remains subject to the Principle of Contradiction, but throws off the yoke of the Principle of Excluded Third … (MS 678:34, late 1910)

The meaning of these claims is quite obvious once we recognize that Peirce had in mind PEM and PC rather than LEM and LNC. It is not the case that, for any proposition of the form “S must be P,” either it or its internal negation (“S must be not-P”) is true; for some propositions expressing necessity, both they and their internal negations are false (e.g., “The Secretary of State must be male” and “The Secretary of State must be non-male”).[13] Here Peirce was re-conceiving PEM as a principle, not simply about propositions with individual subject-terms, but also about propositions that do not express necessity, something along the lines of:

> For any (object-individual) proposition that does not express necessity, either that proposition or its internal negation is true.

Peirce’s claim that PC does not apply to assertions of possibility means only that it is not the case that, for any such proposition, either it or its internal negation is false. For some propositions expressing possibility, they and their internal negations are both true (“The President of the United States may be from Texas” and “The President of the United States may not be from Texas.”) Here Peirce was re-conceiving PC as a principle, not simply about propositions with definite subject-terms, but also about propositions that do not express possibility, something along the lines of:

> For any (object-definite) proposition that does not express possibility, that proposition and its internal negation are not both true.

“Does not apply to” VS. “Is false with regard to”

On Peirce’s view, there is an important difference between saying that PEM or PC does not *apply* to a proposition, and saying that PEM or PC is *false* with regard to a proposition:

> … I do not say that the Principle of Contradiction is *false* of Indefinites. It could not be so without applying to them which is precisely what I deny of it. An argument against what I say, namely, that the Principle of Contradiction does not apply to “A man” because “A man is tall” and “A man is not tall,” can only amount to saying that *that* man that is tall is not, while tall, not tall. That is true; and that is what I mean by refusing to say that the Principle of Contradiction is *false* of “A man” but when it is said of that man that is tall, that he is not not-tall, this is said of the existing man, which is not Indefinite, but is, on the contrary, a certain man and no other. (MS 641:24 2/3 - 3/4, 1909)

PC is not false with regard to object-indefinite propositions, since that principle can only be false with regard to propositions to which it applies, and it applies only to propositions with definite subject-terms. To say that PC is false with regard to “S is P” is to imply (i) that “S” is definite and (ii) that “S is P” is both true and false.

It is reasonable to think that Peirce’s position was analogous with regard to PEM. To say that PEM *is false* with regard to “S is P” is to imply (i) that “S” is individual and (ii) that “S is P” is neither true nor false. Since PEM only applies to propositions with individual (non-general) subject-terms, and since PEM can only be false with regard to propositions to which it applies, it is incorrect to say that the principle is *false* with regard to object-general propositions.

The distinction between a logical principle not applying to a proposition and being false with regard to a proposition is essential for a correct understanding of comments Peirce made about PEM and PC in the context of his experiments with three-valued logical operators (see Lane, 2001). Late in his life, he came to have doubts about the principle of bivalence and defined several three-valued operators, presumably in an attempt to accommodate within formal logic propositions that are neither true nor false. In the following passage, taken from the pages in his logic notebook in which he recorded his work on three-valued logic, Peirce also called into question the principle of excluded middle:

> Triadic logic is that logic which, though not rejecting entirely the Principle of Excluded Middle, nevertheless recognizes that every proposition, S is P, is either true, or false, or else S has a lower mode of being such that it can neither be determinately P, nor determinately not-P, but is at the limit between P and not P. (MS 339, February 23, 1909)

This was echoed in a letter to William James, written only three days later:

> I have long felt that it is a serious defect in existing logic that it takes no heed of the *limit* between two realms. I do not say that the Principle of Excluded Middle is downright *false*; but I *do* say that in every field of thought whatsoever there is an intermediate ground between *positive assertion* and *positive negation* which is just as Real as they. (NEM 3:851, Feb. 26, 1909)

These passages suggest that he did indeed mean by “principle of excluded middle” what contemporary philosophers mean by “law of excluded middle” (LEM):

p V ~p

It is this theorem of classical logic, after all, that fails to be a theorem in many (but not all) modern systems of three-valued logic (See Rescher, 1969, p. 148ff).

But if we keep in mind the “does not apply to” / “is false with regard to” distinction, we can see how the above-quoted comments are compatible with my claim that by “principle of excluded middle” Peirce meant PEM rather than LEM. Peirce did not claim that PEM does *not apply* to propositions which take his third truth value. Rather, he thought that the assignment of a value other than “true” or “false” to a proposition required that PEM be weakened or qualified in some way. But triadic logic would only require a weakening or qualification of PEM if it were intended to accommodate propositions *to which PEM applies in the first place*.

And in fact, the textual evidence strongly suggests that Peirce intended his three-valued connectives to accommodate propositions to which PEM applies but with regard to which PEM is false. Again, if PEM is false with regard to “S is P,” then (i) “S” refers to an individual and (ii) “S is P” is neither true nor false. So the individual to which “S” refers neither has nor lacks the property represented by “P.” And this is in harmony with what Peirce said about the neither-true-nor-false propositions which he apparently hoped to accommodate with his three-valued connectives:

> S has a lower mode of being such that it can neither be determinately P, nor determinately not-P, but is at the limit between P and not P. (MS 339, February 23, 1909)

Peirce was motivated to develop three-valued connectives to accommodate propositions, not to which PEM fails to apply, but to which PEM applies and with regard to which it is false. The weakening or qualification of PEM that triadic logic requires is simply the acknowledgment that, with regard to some propositions to which PEM applies (viz. object-individual, non-modal propositions), that principle is false, i.e., some such propositions take a truth value other than “true” or “false.” This is why, even though Peirce held triadic logic to require a weakening of PEM, his claim that PEM does not

apply to a proposition (e.g., object-general propositions, or propositions expressing necessity) does not imply that the proposition is neither true nor false.[14]

References

Aristotle. (1984). *The Complete Works of Aristotle: The Revised Oxford Translation* (J. Barnes, Ed.). Princeton, NJ: Princeton University Press.

Blackburn, S. (1994). *The Oxford Dictionary of Philosophy*. New York, NY: Oxford University Press.

Chiasson, P. (2001). Peirce’s Logic of Vagueness. In M. Bergman & J. Queiroz (Eds.), *The Commens Encyclopedia: The Digital Encyclopedia of Peirce Studies*. New Edition. Pub. 121220-1909a. Retrieved from http://www.commens.org/encyclopedia/article/chiasson-phyllis-peirce%E2%80%99s-logic-vagueness.

Haack, S. (1974). *Deviant Logic. Some Philosophical Issues*. Cambridge: Cambridge University Press.

Haack, S. (1978). *Philosophy of Logics*. Cambridge: Cambridge University Press.

Hilpinen, R. (1995). Peirce on Language and Reference. In K. Ketner (Ed.), *Peirce and Contemporary Thought: Philosophical Inquiries*. New York, NY: Fordham University Press.

Hughes, G. (1996). Excluded middle, law of. In T. Mautner (Ed.), *A Dictionary of Philosophy*. Cambridge, MA: Blackwell.

Kirwan, C. (1995). Excluded middle, law of. In T. Honderich (Ed.), *The Oxford Companion to Philosophy*. New York, NY: Oxford University Press.

Lane, R. (1997). Peirce’s ‘Entanglement’ with the Principles of Excluded Middle and Contradiction. *Transactions of the Charles S. Peirce Society, 33, 680-703.*

Lane, R. (1998). *Charles Sanders Peirce and the Principle of Bivalence* (Unpublished doctoral dissertation). University of Miami.

Lane, R. (1999). Peirce’s Triadic Logic Revisited. *Transactions of the Charles S. Peirce Society, 35, 284-311.*

Lane, R. (2001). Triadic Logic. In M. Bergman & J. Queiroz (Eds.), *The Commens Encyclopedia: The Digital Encyclopedia of Peirce Studies. New Edition*. Pub. 140730-2104a. Retrieved from http://www.commens.org/encyclopedia/article/lane-robert-triadic-logic.

Marcus, R. (1995). Non-contradiction, principle of. In T. Honderich (Ed.), *The Oxford Companion to Philosophy*. New York, NY: Oxford University Press.

Mautner, T. (1996). Non-contradiction, law of. In T. Mautner (Ed.), *A Dictionary of Philosophy*. Cambridge, MA: Blackwell.

Purtill, R. (1995). Principle of contradiction. In R. Audi (Ed.), *The Cambridge Dictionary of Philosophy*. New York, NY: Cambridge University Press.

Purtill, R. (1995b). Principle of excluded middle. In R. Audi (Ed.), *The Cambridge Dictionary of Philosophy*. New York, NY: Cambridge University Press.

Rescher, N. (1969). *Many-valued Logic*. New York, NY: McGraw-Hill.

Sainsbury, M. (1995). Philosophical Logic. In A. Grayling (Ed.), *Philosophy: A Guide Through the Subject*. New York, NY: Oxford University Press.

Skidmore, A. (1980). On Peirce’s Denial of the Law of Contradiction. *Philosophical Topics*. Supplement, 101-107.

Williamson, T. (1994). *Vagueness*. New York, NY: Routledge.

Endnotes

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1. For uses of “law of excluded middle” to mean something like “Every instance of ‘p or not-p’ is true,” see Kirwan (1995:257), Sainsbury (1995:81), and Purtill (1995b). ↩
2. Haack (1978:246 and 244) uses “excluded middle” and “principle of non-contradiction” to refer to the familiar theorems of classical logic, the formulae
   p V ~p and ~(p & ~p). ↩ ↩
3. For uses of “law of excluded middle” and “law of non-contradiction” to refer to this sort of natural language analogue of theorems of classical logic, see Williamson (1994:9), Blackburn (1994:129 and 81), Hughes (1996:140) and Mautner (1996:295). ↩ ↩
4. For uses of “principle of non-contradiction” to mean something like “Every instance of ‘p and not-p’ is false,” see Marcus (1995:625) and Purtill (1995). ↩
5. Recall Aristotle’s “most certain of all” principles: “the same attribute cannot at the same time belong and not belong to the same subject in the same respect” (Metaphysics 1005b 19-20), and his claim that “there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate.” (Metaphysics 1011b 23-25) ↩
6. Peirce stated his principle of excluded middle in the material mode at 1.434 (c.1896):
   … the individual is determinate in regard to every possibility, or quality, either as possessing it or as not possessing it. This is the principle of excluded middle, which does not hold for anything general, because the general is partially indeterminate …
   and in the formal mode at MS 611:13 (1908):
   By the Principle of Excluded Middle (or of excluded third,) is always meant the principle that no pair of mutually contradictory predicates are both false of any individual subject.

(Of course, to say that the twelve disciples of Jesus were all apostles or were not apostles are both false.) ↩

7. Peirce stated his principle of contradiction in the formal mode at MS 611:12-3 (1908): “By the Principle of Contradiction, accurate writers for nearly two centuries have understood the principle that a pair of contradictory predicates, such as “is P” and “is not P” (or other than every P) are both true only of Nothing, and not of any definite subject.” ↩
8. Object-indeterminate propositions have subject phrases which include what Peirce called selective pronouns, two sorts of which he says are especially important in logic: universal selectives and particular selectives. Universal selectives include “any, every, all, no, none, whatever, whoever, everybody, anybody, nobody.” (2.289, c.1893) The use of universal selectives results in generality, as in the general propositions “Nobody cares,” “Every good boy deserves fudge,” and “All zebras are striped.” Particular selectives include “some, something, somebody, a, a certain, some or other, a suitable, one.” (Ibid.) The use of particular selectives results in indefiniteness, as in the indefinite propositions “Somebody had better explain,” “One day I’ll escape from prison,” and “A possum stole my cornbread.” (Singular subject-terms, including names and definite descriptions, do not include selectives.)

   So, in more modern terms, general propositions are represented in predicate calculus as universally quantified formulae, and indefinite propositions are represented in predicate calculus as existentially quantified formulae. However, some of Peirce’s own examples of generality, e.g., “Man is mortal,” do not include selective pronouns, so it is perhaps somewhat inaccurate to identify, as do Haack (1974:109) & Hilpinen (1995:292), Peirce’s general object-indeterminacy with universal quantification. Although general propositions are represented in predicate calculus with a universal quantifier, not every general proposition in natural language need contain a quantifier phrase. ↩
9. Some commentators have run afoul of the distinction between indefiniteness and fuzziness, taking Peirce to have denied the principle of contradiction (in some sense) for statements containing borderline terms. For example, Skidmore says:

   It may strike one as somewhat curious that Peirce has chosen violation of (LC) [the law of contradiction] as the defining characteristic of vagueness. For it seems intuitively clearer to understand as vague something to which the law of excluded middle (EM) does not apply. (1980:105)

   In this respect, see also Williamson (1994:51-52). ↩
10. Of course, this is not a sufficient condition of individuality, since for some general (non-individual) subject-terms “S”, either “S is P” or “S is not-P” is true (“All U.S. Presidents elected before 2000 are male,” “All squares are four-sided figures,” “All the bills in my

wallet are non-counterfeit” … etc.) Prima facie this conclusion is in tension with the fact that Peirce “defined” the individual as that to which PEM applies (PPM:175, 1903) and “defined” the general (i.e., the non-individual) as that to which PEM does not apply (5.448, 1905). PEM gives only a necessary condition, not both necessary and sufficient conditions, for individuality, so it might be thought that when Peirce claimed to define individuality in terms of PEM, he was simply overstating his case.

But a more charitable interpretation will appeal to the fact that, in many of his statements of PEM, Peirce simply stipulated that the principle applies to all and only individuals, i.e., he “built into” his statement of PEM the claim that it does not apply with regard to general subjects. If this is correct, then Peirce’s definitions of the individual as that to which PEM applies, and of the general as that to which PEM does not apply, are definitions, strictly speaking, although not very informative ones. There is an analogous understanding of Peirce’s tendency to define “vagueness” (indefiniteness) in terms of PC. ↩

11. For much more on Peirce’s principles of excluded middle and contradiction, including an account of Peirce’s views on generality and vagueness of propositional predicates and explanations of passages by Peirce that seem not to support the reading set forth above, see Lane (1997) and (1998, ch.1, 2 and 3). ↩
12. Although most of Peirce’s discussions of modal expressions were couched in terms of modal propositions (e.g., 2.323, 1902; 2.383, 1902; 6.370, 1902; NEM 3:813, 1905), Peirce’s denials of the applicability of PEM and PC were phrased in terms of “assertions of modality”; that is, he wrote that PEM does not apply to “assertions of necessity” and that PC does not apply to “assertions of possibility.” (MS 678:27ff., 1910) The phrases “assertion of necessity” and “assertion of possibility” are less likely to mislead than the phrases “necessary proposition” and “possible proposition.” “Necessary proposition” suggests a proposition that is necessarily true; but by “assertion of necessity” Peirce means an assertion of a proposition having one of the following forms: “S must be P”, “S shall be P”, “S would be P”, and “It is necessary that S is P”; and it is to propositions of these forms, not to necessarily true propositions, that Peirce held PEM not to apply.

    Similarly, “possible proposition” suggests a proposition the truth of which is possible (as opposed to a necessarily false, or impossible, proposition); but by “assertion of possibility,” Peirce meant an assertion of a proposition having one of the following forms: “S can be P”, “S may be P”, “S might be P”, and “It is possible that S is P”; and it is to propositions of these forms, not to all possibly true propositions, that Peirce held PC not to apply. Since it is really the logical structure of propositions with which Peirce was concerned, and since I am primarily concerned with Peirce’s alleged claims about the truth values of specific sorts of propositions, I have chosen to phrase my discussion in terms of propositions. But I refer,

not to necessary propositions and possible propositions, but to propositions expressing necessity and propositions expressing possibility; and I use “modal proposition” as the broader phrase covering both sorts of proposition. ↩

13. By “internal negation” I mean merely the result of negating the predicate of a proposition rather than the entire proposition. The internal negation of “S is P” is “S is non-P”; the internal negation of “S must be P” is “S must be non-P”, etc. ↩
14. For a more detailed account of the connections between Peirce’s triadic logic and the principles of excluded middle and contradiction, see Lane (1999). ↩