Pseudo-continuum

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...I define a *pseudo-continuum* as that which modern writers on the theory of functions call a continuum. But this is fully represented by, and according to G. Cantor stands in one-to-one correspondence with, the totality of real values, rational and irrational; and these are iconized, in their turn, according to these writers [by the] entire body of decimal expressions carried out to the right to all finite powers of 1/10 without going on to Cantor's ωth place of decimals.

For it is a principle continually employed in the reasoning of the universally accepted "doctrine of limits" that two values, that differ at all, differ by a *finite value*, which would not be true if the ωth place of decimals were supposed to be included in their exact expressions; and indeed the whole purpose of the doctrine of limits is to avoid acknowledging that that place is concerned. Consequently the denumeral rows of figures which, by virtue of a simple general principle, are in one-to-one correspondence with the values, have relations among themselves, quite regardless of their denoting those values that perfectly agree in form with the relations between the values; and consequently these unlimited decimal fractions themselves, apart from their significations, constitute a pseudo-continuum. This consideration renders it easy to define a pseudo-continuum. It is in the first place a collection of objects absolutely distinct from one another. Now from the fact that Cantor and others call it a "continuum," as well as from other things they say about it, I am led to suspect that they do not regard the pseudo-continuum of unlimited decimal expressions as [having members] all absolutely distinct from any other, for the reason that, taking any one of them, it does not possess any one elementary and definite non-relative character which is not possessed by any other of them. But this is not what I mean, nor what is generally meant, by a collection of absolutely independent members. What I mean by that expression is that every member is distinguished from every other by possessing some one or another elementary and definite non-relative character which that other does not possess; and that this is the usual acceptation of the expression is evidenced by the fact that the majority of logicians are in the habit of conceiving of a universe of absolutely distinct individual objects, by which they only mean that every individual is in every respect, of a certain universe of respects, determined in one or other of two ways and that every individual is differently determined from every other in some of those respects; and they do not generally conceive that every individual object has a determination in any one elementary and definite respect, while all the other individuals are determined in the opposite way.