

# Probability

1878 | The Doctrine of Chances | W 3:280-1; CP 2.649-50

As Locke says, the probable argument is “such as for the most part carries truth with it.”

According to this, that real and sensible difference between one degree of probability and another, in which the meaning of the distinction lies, is that in the frequent employment of two different modes of inference, one will carry truth with it oftener than the other. It is evident that this is the only difference there is in the existing fact. Having certain premisses, a man draws a certain conclusion, and as far as this inference alone is concerned the only possible practical question is whether that conclusion is true or not, and between existence and non-existence there is no middle term. [—] For we found that the distinction of reality and fiction depends on the supposition that sufficient investigation would cause one opinion to be universally received and all others to be rejected. That presupposition, involved in the very conceptions of reality and figment, involves a complete sundering of the two. It is the heaven-and-hell idea in the domain of thought. But, in the long run, there is a real fact which corresponds to the idea of probability, and it is that a given mode of inference sometimes proves successful and sometimes not, and that in a ratio ultimately fixed. As we go on drawing inference after inference of the given kind, during the first ten or hundred cases the ratio of successes may be expected to show considerable fluctuations; but when we come into the thousands and millions, these fluctuations become less and less; and if we continue long enough, the ratio will approximate toward a fixed limit. We may, therefore, define the probability of a mode of argument as the proportion of cases in which it carries truth with it.

1903 | Harvard Lectures on Pragmatism: Lecture VI | EP 2:215; CP 5.169

Probability applies to the question whether a specified kind of event will occur when certain predetermined conditions are fulfilled; and it is the ratio of the number of times in the long run in which that specified result would follow upon the fulfillment of those conditions to the total number of times in which those conditions were fulfilled in the course of experience. It essentially refers to a course of experience, or at least of real events; because mere possibilities are not capable of being counted.

1910 | Note (Notes on Art. III) [R] | CP 2.664-5

...the kind of reasoning which creates likelihoods by virtue of observations may render a likelihood *practically* certain – as certain as that a stone let loose from the clutch will, under circumstances not obviously exceptional, fall to the ground – and this conclusion may be that under a certain general condition, easily verified, a certain actuality will be *probable*, that is to say, will come to pass once in so often in the long run. One such familiar conclusion, for example, is that a die thrown from a dice box will with a *probability* of one-third, that is, once in three times in the long run, turn up a number (either *tray* or *size*) that is divisible by three. But this can be affirmed with practical certainty only if by a “long run” be meant an endless series of trials, and (as just said) infinity divided by infinity gives of itself an

entirely indefinite quotient. It is therefore necessary to define the phrase. I might give the definition with reference to the probability,  $p$ , where  $p$  is any vulgar fraction, and in reference to a generic condition,  $m$ , and a specific kind of event  $n$ . But I think the reader will follow me more readily, if in place of the letter,  $m$  (which in itself is but a certain letter, to which is attached a peculiar meaning, that of the fulfillment of some generic condition) I put instead the supposition that a die is thrown from a dice box; and this special supposition will be as readily understood by the reader to be replaceable by any other general condition along with a simultaneous replacement of the *event*, that a number divisible by three is turned up, and at the same time with the replacement of one third by whatever other vulgar fraction may be called for when some different example of a probability is before us. I am, then, to define the meanings of the statement that the *probability*, that if a die be thrown from a dice box it will turn up a number divisible by three, is one-third. The statement means that the die has a certain "would-be"; and to say that a die has a "would-be" is to say that it has a property, quite analogous to any *habit* that a man might have. Only the "would-be" of the die is presumably as much simpler and more definite than the man's habit as the die's homogeneous composition and cubical shape is simpler than the nature of the man's nervous system and soul; and just as it would be necessary, in order to define a man's habit, to describe how it would lead him to behave and upon what sort of occasion – albeit this statement would by no means imply that the habit *consists* in that action – so to define the die's "would-be," it is necessary to say how it would lead the die to behave on an occasion that would bring out the full consequence of the "would-be"; and this statement will not of itself imply that the "would-be" of the die *consists* in such behavior.

Now in order that the full effect of the die's "would-be" may find expression, it is necessary that the die should undergo an endless series of throws from the dice box, the result of no throw having the slightest influence upon the result of any other throw, or, as we express it, the throws must be *independent* each of every other.

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1910 [c.] | Letters to Paul Carus | CP 8.225

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None of the books contain a definition of mathematical probability (which is what I mean by "probability" however measured) which will hold water. For the sake of simplicity, I will define it in a particular example. If, then, I say that the probability that if a certain die be thrown in the usual way it will turn up a number divisible by 3 (i.e., either 3 or 6) is  $1/3$ , what do I mean? I mean, of course, to state that that die has a certain habit or disposition of behaviour in its present state of wear. It is a would be and does not consist in actualities or single events in any multitude finite or infinite. Nevertheless a habit does consist in what *would* happen under certain circumstances if it should remain unchanged throughout an endless series of actual occurrences. I must therefore define that habit of the die in question which we express by saying that there is a probability of  $1/3$  (or odds of 1 to 2) that if it be thrown it will turn up a number divisible by 3 by saying how it would behave if, while remaining with its shape, etc. just as they are now, it were to be thrown an endless succession of times. Now it is very true that it is quite impossible that it should be thrown an infinite succession of times. But this is no objection to my supposing it, since that impossibility is merely a physical, or if you please, a metaphysical one, and is not due to any logical impossibility to the occurrence in a finite time of an endless succession of events each occupying a finite time. For when Achilles overtook the tortoise he had to go through such an endless series (*endless in the series*, but not *endless in time*) and supposedly actually did so.